**Code No: 152AA** **R18**

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD**

**B. Tech I Year II Semester Examinations, September - 2023**

**MATHEMATICS-II**

**(Common to EEE, CSE, IT, CSIT, ITE, CE(SE), CSE(CS), CSE(DS), CSE(N), CSD)**

**Time: 3 Hours** **Max. Marks: 75**

**Note:** i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

**PART – A**

**(25 Marks)**

1.a) Solve $(1+x^2)\frac{dy}{dx} + y = \tan^{-1} x$. [2]

b) Solve $(y+x)dx = (y-x)dy$. [3]

c) Define Legendre's linear differential equation of n$^{th}$ order. [2]

d) Solve $\frac{d^4x}{dt^4} + 4x = 0$. [3]

e) Write the formulae to find the centre of gravity of a plane lamina whose density at point (x, y, z) is $x^2y+z$. [2]

f) Evaluate $\int_0^1\int_1^2\int_2^3 (x+y+z)\,dxdydz$. [3]

g) Find the maximum value of the directional derivative of $\emptyset = x^2yz$ at (1, 4, 1). [2]

h) Prove that $\overline{f} = (x^2+xy^2)i + (y^2+x^2y)j$ is conservative and find its scalar potential. [3]

i) Is the work done by a force in moving a particle from one point to another point in an irrotational field is independent of the path of integration? Justify the answer. [2]

j) What is the result when Stoke's theorem is applied to a closed surface? [3]

**PART - B**

**(50 Marks)**

2.a) Solve $\frac{dy}{dx} + \frac{y}{x} = y^2 \log x$.

b) Solve $(x^2-ay)dx + (y^2-ax)dy = 0$. [5+5]

**OR**

3.a) The number $N$ of bacteria in a culture grows at a rate proportional to $N$. The value of $N$ was initially 100 and increased to 332 in one hour. What was the value of $N$ after $1\frac{1}{2}$ hour?

b) Solve $y = 2px + y^2p^3$, $p = \frac{dy}{dx}$. [5+5]

4.a) Solve by the method of variation of parameters $(D^2 - 2D)y = e^x \sin x$.

b) Solve $(D^3 - 1)y = e^x$. [5+5]

**OR**

5. Solve $y'' + 4y' + 4y = 4\cos x + 3\sin x;\ y(0) = 1, y'(0) = 0$. [10]

6.a) Change the order of integration and solve $\int_0^c \int_{x^2/a}^{2a-x} xy^2 dydx$

b) Find the volume bounded by the cylinders $x^2 + y^2 = 4$ and $z = 0, z = 5$. [5+5]

**OR**

7. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$ by changing to spherical coordinates. [10]

8.a) Prove that $div\left(\frac{\vec{r}}{r^3}\right) = 0$, where $\vec{r} = xi + yj + zk$, $r = |\bar{r}|$.

b) Prove that $div.(grad\, r^m) = m(m+1)r^{m-2}$, $r = |x\bar{i} + y\bar{j} + 2\bar{k}|$. [5+5]

**OR**

9.a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, -1,2).

b) Prove that if $\vec{r}$ is the position vector of any point in space then $r^n\vec{r}$ is irrotational and is solenodial if n = -3. [5+5]

10. Evaluate $\iint_S \bar{F}.\bar{n}dS$ where $\bar{F} = 2x^2y\mathrm{i} - y^2\mathrm{j} + 4xz^2\mathrm{k}$ and $S$ is the closed surface of the region in the first octant by the cylinder $y^2 + z^2 = 9$ and the planes $x = 0$, $x = 2$, $y = 0$, $z = 0$. [10]

**OR**

11. Verify Green's theorem for $\bar{F} = (xy + x^2)\bar{i} + (x^2 + y^2)\bar{j}$ along a closed curve C of the region bounded by $y = x$ and $y = x^2$. [10]

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