

**R18**

Code No: 152AA

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year II Semester Examinations, September - 2023

MATHEMATICS-II

(Common to EEE, CSE, IT, CSIT, ITE, CE(SE), CSE(CS), CSE(DS), CSE(N), CSD)

Time: 3 Hours

Max. Marks: 75

**Note:** i) Question paper consists of Part A, Part B.

ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.

iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

**PART - A****(25 Marks)**

- 1.a) Solve  $(1 + x^2) \frac{dy}{dx} + y = \tan^{-1} x$ . [2]
- b) Solve  $(y + x)dx = (y - x)dy$ . [3]
- c) Define Legendre's linear differential equation of  $n^{\text{th}}$  order. [2]
- d) Solve  $\frac{d^4x}{dt^4} + 4x = 0$ . [3]
- e) Write the formulae to find the centre of gravity of a plane lamina whose density at point  $(x, y, z)$  is  $x^2y+z$ . [2]
- f) Evaluate  $\int_0^1 \int_1^2 \int_2^3 (x+y+z) dx dy dz$ . [3]
- g) Find the maximum value of the directional derivative of  $\phi = x^2yz$  at  $(1, 4, 1)$ . [2]
- h) Prove that  $\vec{f} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$  is conservative and find its scalar potential. [3]
- i) Is the work done by a force in moving a particle from one point to another point in an irrotational field is independent of the path of integration? Justify the answer. [2]
- j) What is the result when Stoke's theorem is applied to a closed surface? [3]

**PART - B****(50 Marks)**

- 2.a) Solve  $\frac{dy}{dx} + \frac{y}{x} = y^2 \log x$ .
- b) Solve  $(x^2 - ay)dx + (y^2 - ax)dy = 0$ . [5+5]

**OR**

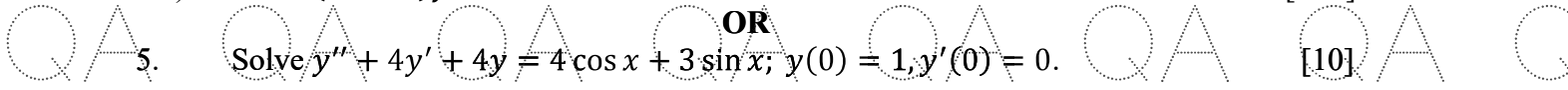
- 3.a) The number  $N$  of bacteria in a culture grows at a rate proportional to  $N$ . The value of  $N$  was initially 100 and increased to 332 in one hour. What was the value of  $N$  after  $1\frac{1}{2}$  hour?
- b) Solve  $y = 2px + y^2 p^3$ ,  $p = \frac{dy}{dx}$ . [5+5]



4.a) Solve by the method of variation of parameters  $(D^2 - 2D)y = e^x \sin x$ .

b) Solve  $(D^3 - 1)y = e^x$ .

[5+5]



5. Solve  $y''' + 4y' + 4y = 4 \cos x + 3 \sin x$ ;  $y(0) = 1, y'(0) = 0$ .

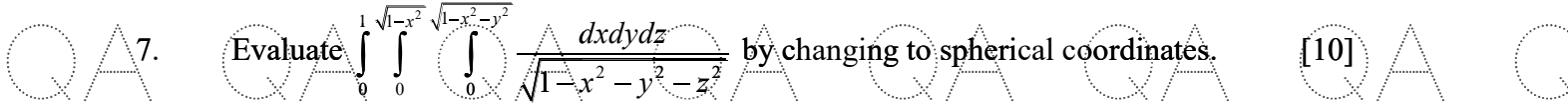
[10]

6.a) Change the order of integration and solve  $\int_0^c \int_{x^2/a}^{2a-x} xy^2 dy dx$

b) Find the volume bounded by the cylinders  $x^2 + y^2 = 4$  and  $z = 0, z = 5$ .

[5+5]

OR



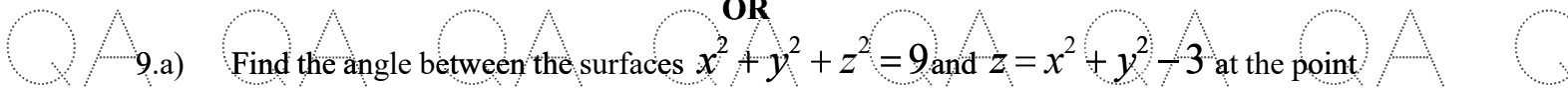
7. Evaluate  $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx dy dz}{\sqrt{1-x^2-y^2-z^2}}$  by changing to spherical coordinates.

[10]

8.a) Prove that  $\text{div} \left( \frac{\vec{r}}{r^3} \right) = 0$ , where  $\vec{r} = xi + yj + zk, r = |\vec{r}|$ .

b) Prove that  $\text{div}(\text{grad } r^m) = m(m+1)r^{m-2}, r = |xi + yj + zk|$ .

[5+5]



9.a) Find the angle between the surfaces  $x^2 + y^2 + z^2 = 9$  and  $z = x^2 + y^2 - 3$  at the point  $(2, -1, 2)$ .

b) Prove that if  $\vec{r}$  is the position vector of any point in space then  $r^n \vec{r}$  is irrotational and is solenoidal if  $n = -3$ .

[5+5]

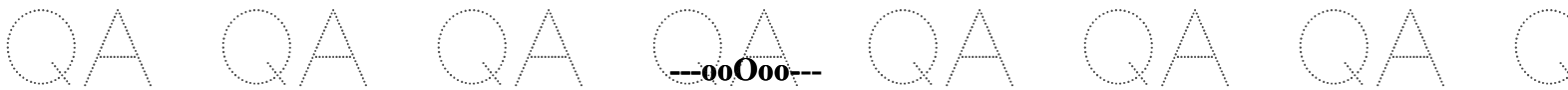
10. Evaluate  $\iint_S \vec{F} \cdot \vec{n} dS$  where  $\vec{F} = 2x^2yi - y^2j + 4xz^2k$  and  $S$  is the closed surface of the region in the first octant by the cylinder  $y^2 + z^2 = 9$  and the planes  $x = 0, x = 2, y = 0, z = 0$ .

[10]

OR

11. Verify Green's theorem for  $\vec{F} = (xy + x^2)\vec{i} + (x^2 + y^2)\vec{j}$  along a closed curve  $C$  of the region bounded by  $y = x$  and  $y = x^2$ .

[10]



---ooOoo---

